Matrices in 3-space of Magic Squares

An $m\times n$ matrix A is a rectangular array of mn numbers arranged in m rows and n columns. If a_{ij} is the element in the i:th and the j:th column, then the Magic Square looks like ordinary matrices:

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

[11]

If assume the case that $A=(a_{ij})$ into a range i from 1 to m and j from 1 to n. If $m = n$ we say that A is a square matrix and if also the main diagonals from a_{11} to a_{mn} and a_{1n} to a_{m1} are arranged in the same pattern like the rows a_{11} to a_{1n} and the columns a_{11} to a_{1n} it has condition to Magic Squares. The elements a_{ij} of the matrices we use in this Compendium will always be real Integer number.

The transpose of an $m\times n$ matrix A is then the $n \times m$ matrix A^T whose rows are the columns of A:

$$A^T=\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2m} & \cdots & a_{mn} \end{pmatrix}$$

[11]

Matrix A is called symmetric if $A^T = A$. Symmetric matrices are necessarily into Magic Squares. Take into consideration that $(A^T)^T = A$ for every matrix A into Magic Squares. Examples of $n = 5$.

$$A \quad B \quad C \quad D \quad E$$

$$A = \begin{matrix} & 2nd & \\ 1st & B & \dots \\ & \vdots & \end{matrix} \quad \rightarrow \quad A^T = \begin{matrix} & 1st & \\ 2nd & B & \dots \\ & \vdots & \end{matrix}$$

If take the second letter in the alphabet, B, and then count in the row every 2nd and in the column every 1st letter. Then we get the matrix A of letter combination and if transpose we get matrix A^T.

$$A=\begin{pmatrix} B & D & A & C & E \\ C & E & B & D & A \\ D & A & C & E & B \\ E & B & D & A & C \\ A & C & E & B & D \end{pmatrix} \quad \rightarrow \quad A^T=\begin{pmatrix} B & C & D & E & A \\ D & E & A & B & C \\ A & B & C & D & E \\ C & D & E & A & B \\ E & A & B & C & D \end{pmatrix}$$

The algorithm of the letter is always that the first arrays of letter in the first row will take the first arrays of letter in the first column. Ex. A, letter D in the row will get letter E into its own column. If multiplied two matrices, then the same *grad*- in 3-spaces: $2\times3\ 3\times4 = 2\times4$ thus: $5\times5\ 5\times5 = 5\times5$.

The algorithm for higher prime order of Magic Square

When an $m \times n$ matrix A is of order $n = 3$, there exist no panmagic square solutions or say of any order $4k + 2$, where k is an Integer. That will say no panmagic squares of order $n = 3, 6, 10, 14\ldots$ But there exist panmagic squares when n belongs to an odd Integer that is a prime Integer, that will say order of $n = 5, 7, 11, 13, 17, 19, 23, 27, 29$ *and* 31. The algorithm for these orders is of a very simple construction and I believe that the nature has approximately exact same process in the protein syntheses. The panmagic squares are then constructed with one part of letter and one part of Integer, and when overlapping both parts, we have the magic squares solved with the key.

$$\begin{array}{ccc} & 2nd & \\ 1st & B & \ldots \\ & \vdots & \end{array}$$

This symbol indicates that the first letter in the left upper corner will be B. The horizontal rows will have every second letter in the alphabetic order of $A\ B\ C\ D\ E$ in a panmagic square of $n = 5$. The first vertical column will have every letter after B in alphabetic order of $A\ B\ C\ D\ E$. We have

$$\begin{array}{ccccc} B & D & A & C & E \\ C & \vdots.. & \vdots.. & \vdots.. & \vdots.. \\ D & \vdots.. & \vdots.. & \vdots.. & \vdots.. \\ E & \vdots.. & \vdots.. & \vdots.. & \vdots.. \\ A & \vdots.. & \vdots.. & \vdots.. & \vdots.. \end{array} \rightarrow \begin{array}{ccccc} B & D & A & C & E \\ C & E & \vdots.. & \vdots.. & \vdots.. \\ D & A & \vdots.. & \vdots.. & \vdots.. \\ E & B & \vdots.. & \vdots.. & \vdots.. \\ A & C & \vdots.. & \vdots.. & \vdots.. \end{array} \rightarrow \begin{array}{ccccc} B & D & A & C & E \\ C & E & B & \vdots.. & \vdots.. \\ D & A & C & \vdots.. & \vdots.. \\ E & B & D & \vdots.. & \vdots.. \\ A & C & E & \vdots.. & \vdots.. \end{array} \rightarrow \begin{array}{ccccc} B & D & A & C & E \\ C & E & B & D & \vdots.. \\ D & A & C & E & \vdots.. \\ E & B & D & A & \vdots.. \\ A & C & E & B & \vdots.. \end{array}$$

In the second matrix first row at D we add from the first matrix column under D. In the third matrix first row at A we add the first matrix column under A. In the fourth matrix first row at C we add the first matrix column under C. Next step is to add the matrix at first row under letter E.

$$\begin{array}{ccccc} B & D & A & C & E \\ C & E & B & D & A \\ D & A & C & E & B \\ E & B & D & A & C \\ A & C & E & B & D \end{array} + \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & \vdots.. & \vdots.. & \vdots.. \\ 5 & \vdots.. & \vdots.. & \vdots.. & \vdots.. \\ 2 & \vdots.. & \vdots.. & \vdots.. & \vdots.. \\ 4 & \vdots.. & \vdots.. & \vdots.. & \vdots.. \end{array} \rightarrow \begin{array}{ccccc} B1 & D2 & A3 & C4 & E5 \\ C3 & E4 & B & D & A \\ D5 & A & C & E & B \\ E2 & B & D & A & C \\ A4 & C & E & B & D \end{array} \rightarrow \begin{array}{ccccc} B1 & D2 & A3 & C4 & E5 \\ C3 & E4 & B5 & D1 & A2 \\ D5 & A1 & C2 & E3 & B4 \\ E2 & B3 & D4 & A5 & C1 \\ A4 & C5 & E1 & B2 & D3 \end{array}$$

When adding Integer the algorithm is that the diagonals from upper right to down left the Integer must always go in a pattern of an increasing sequence of Integer or a pattern of a decreasing of Integer. This law could not be broken at any matrix in order of odd Integer in a primed sequence of order. The letter arrays are also going in alphabetic diagonal pattern sequences. If investigate the higher order of primed Magic Squares then it's possibly to see that in the nature process like in the protein synthesis or construction of *Exons* in the DNA, which are the protein coding genes and make-up of only 3% of the human genome, the main diagonal from upper left to lower right must be mathematically magic. Otherwise it will be too easy to construct the algorithm in nature process. This is possibly to see in the alphabetic letter algorithm and in some Magic Squares it's possibly to see that an Integer in a array should not arise two times in the same rows or columns. If take this nature algorithm of alphabetic letter and Integer and put it in squares, it will be magic.

The key into prime Magic Squares

Prime number p is one positive Integer having exactly one positive divisor other than 1, meaning it's an Integer that cannot be factored. It will be possibly to proof pure primed Magic Squares if the start value a is corresponding to the algorithm $5k+2$, where k belongs to an odd Integer from a sequences $k = 1, 3, 5, 7, 9, 11 \ldots\ldots$. The delta difference between terms d is equal to constant with value $d = 50$. The algorithm prime starts with one start value a and sequence with adding d.

$$a = 5k + 2 = 7$$

The lowest prime Integer of start value is $a = 7$, because the following sequence must be prime.

3 5 7 11 13 17 19 23 27 29 31 37 41 43 47 51 53 57 59 61 63 67 69 71 73 77 79 81 83 87 …

$\Delta\, d = 50$ $\Delta\, d = 50$

The Key

$A1$	=	7
$A2$	=	57
$A3$	=	107
$A4$	=	157
$B1$	=	207
$B2$	=	257
$B3$	=	307
$B4$	=	357
$C1$	=	407
$C2$	=	457
$C3$	=	507
$C4$	=	557
$D1$	=	607
$D2$	=	657
$D3$	=	707
$D4$	=	757

$A1$	$B3$	$C4$	$D2$	7	307	557	657
$D4$	$C2$	$B1$	$A3$	757	457	207	107
$B2$	$A4$	$D3$	$C1$	257	157	707	407
$C3$	$D1$	$A2$	$B4$	507	607	57	357

$A1$	$B4$	$C2$	$D3$	7	357	457	707
$C3$	$D2$	$A4$	$B1$	507	657	157	207
$D4$	$C1$	$B3$	$A2$	757	407	307	57
$B2$	$A3$	$D1$	$C4$	257	107	607	557

Here are two examples of pure prime Magic Squares. However, with the algorithm of start value $a = 5k+2$ will it be possibly to find more true solutions of primed Magic Squares of order $n = 4$.

$$\Sigma(n;a,d) = \frac{1}{2} \cdot n \cdot \left[2 \cdot a + d \cdot \left(n^2 - 1 \right) \right] = 1528$$

The magic sum of the primed tropic house of order $n = 4$ is $sum = 1528$. Here with start value a and increasing with the constant d. If now use $k = 3$, then the magic constant will be $sum = 1568$. This algorithm of the primed house will also be possibly to use into magic squares of order $n = 5$

Magic Square of Order 9

The Devils Square of order n=9 with in baked Magic Square of order n=3, year 2022. S1=15 and S1=369.

11	18	13	74	81	76	29	36	31
16	14	12	79	77	75	34	32	30
15	10	17	78	73	80	42	28	35
56	63	58	38	45	40	20	27	22
61	59	57	43	41	39	25	23	21
60	55	62	42	37	44	24	19	26
47	54	49	2	9	4	65	72	67
52	50	48	7	5	3	70	68	66
51	46	53	6	1	8	69	64	71

⌘

11	18	13	76	81	74	29	36	31
16	14	12	75	77	79	34	32	30
15	10	17	80	73	78	42	28	35
56	63	58	40	45	38	20	27	22
61	59	57	39	41	43	25	23	21
60	55	62	44	37	42	24	19	26
47	54	49	4	9	2	65	72	67
52	50	48	3	5	7	70	68	66
51	46	53	8	1	6	69	64	71

The square in blue is of order n3 and it's the only possibly magic square to get on order n3. Here in order n9. There are different main diagonal in one and two on magic square of order n9. That makes different.

The Magic Square of order n9 is composed with 9 different Magic Square of order n3. The blue squares in the center are an original Magic Square of order 3, with magic sum of Σ15. The sum in each square will be like follows:

42	231	96
177	123	69
150	15	204

A2	C3	B1
C1	B2	A3
B3	A1	C2

2	9	4
7	5	3
6	1	8

All 9 squares of order n3 are built up according to the two formulas of magic square n3 above. This will make that each square also has magic main diagonals. Together they will have the magic sum Σ369 of n9. To get the magic square with sum Σ15 of order n3 in the blue area, A1 above must be in the center.

The formulas to get the magic sum of order n of square. Where order n3 will give the magic sum of Σ15.

- S1 = n (n^2 + 1)/2
- S2 = n (n^2 + 1) (2n^2 + 1)/6 = S1*(2n^2 + 1)/3
- S3 = n*S1^2
- S4 = S2*(6n*S1 - 1)/5
- S5 = (3n*S2^2 - S3)/2

Here S1 stands for only magic, S2 for bimagic and S3 for trimagic o.s.v. Simplified it will look like follows:

- S1 = (1/2) n^3 + (1/2) n
- S2 = (1/3) n^5 + (1/2) n^3 + (1/6) n
- S3 = (1/4) n^7 + (1/2) n^5 + (1/4) n^3

Where order n9 will give the magic sum of Σ369, bimagic sum of Σ20049 and trimagic sum of Σ1225449.