Looking for her for million times …
(The process to solve the 16th trimagic square)

Chen Qinwu

It was by chance that I had several talks with Chen Mutian who is a retired professor of our department. From the talks I knew that he had been working at the solution to magic square. In the beginning, I felt such a magic was unimaginable considering its infinitesimal possibility. (Exactly speaking, even if there are 10^{12} trimagic squares in the world, the possibility of existence is still below 10^{-495} because of $256!=8.6\text{x}10^{507}$)

Then Prof. Chen told me that he spent about one year solving 11^{th} order bimagic square. But a German solved 12^{th} order trimagic square three years ago. I asked him what use it was to spend so much energy and time solving it. He said that he derived much pleasure from it, but many experts were finding the significance of its application, the improvement of algorithms, the parallel processing, the scheduling of the economy decision-making, etc. I was doubtful about it. He told me that he was searching for the answer to 16th trimagic square. Owing to the large amount of the calculation, the program often had to be executed for several days, and we should select certain strategy to reduce the amount of the calculation. Recently he divided the program into some sections, and came to the campus to do a check and make some adjustments from six miles away every afternoon. Then I thought the calculational speed would be obviously improved, if I transplanted the program to a complete 32bits environment in the computer of my laboratory with a CPU of Pentium 4 2.0G Hz. As expected, the speed was about twenty times more than the original after we succeeded in transplanting the program. This made us very excited.

In our subsequent contacts, Prof. Chen introduced some effective methods he used to solve magic square. Gradually, I came to have some understanding of the problem. He used a matrix of 16 rows which had been arranged to meet the requirement to make row to row adjustments, so the amount of calculations of a row is $16^{16}\approx1.8\text{x}10^{19}$. Even though the fastest super computer in the world was used to seek the solution, several years were needed. So we had to look for some algorithms to decrease the amount of calculations so that ordinary computers could seek the solution of the problem in limited time.

We used the numbers between 0 and 255 so that the computer could handle easily. After analyzing the construction of the matrix and the characteristics of the problem, I thought there was a most probable way that we could meet the requirement. If I make the sum of two numbers in the same row amount to 255, the amount of calculations will decrease to $16^{8}\approx4.3\text{x}10^{9}$, Which can be solved by the computer in my laboratory in a short time. For the combination of complementary numbers whose sum is 255 has been occupied by columns, if I can make the higher seven bits of the two numbers (binary) reverse and the lowest bit similar (i.e. Num2 = Num1 XOR 0xFE), where the sum of two even numbers is 254 and the sum of two odd numbers is 256, it can meet the above-mentioned requirement and can be transformed by the given matrix above.

I told Prof. Chen the method, and he said I might as well have a try. Of course, because the above-mentioned transformation needed some programs, it was very complicated. So I wrote a program and asked Prof. Chen to execute, I thought there would be a result in two or three days. But Prof. Chen realized the complexity of the problem. He said it was not so easy. Many people have made a sustained

effort and spent a lot of time looking for it, but so far there has been no answer. He encouraged me to have a try.

I went on undeterred by the difficulties ahead. Before such difficulties, I never lost my confidence. On the contrary, the more difficult the problem was, the more courage I had. I inputted the program to the computer and debugged, I found the problem was much more complex than I had expected. After some days' modifying and debugging, there was an initial result. Although there was no solution fulfilling the requirement of all sixteen rows, thousands of rows met the requirement. After modifying again and again, there were eight rows meeting the requirement simultaneously. So I continued writing programs, I separated the number combination of the remaining eight rows to search independently. Because the amount of the calculation had added up to $8^{16} \approx 2.6 \times 10^{14}$, it had not even finished in several days and no another new row meeting the requirement appeared.

I thought it deeply over and over，and programmed to test. I found if I constructed the matrix independently and when the two numbers (whose sum was 255) were combined complementarily, many matrixes of 16 rows meeting the requirement would come out. The program did not even end after running several hours to output millions of matrixes. When the two numbers (i.e. the sum of two even numbers is 254 and the sum of two odd numbers is 256) were combined complementarily, the program could find many matrixes which had 12 rows meeting the requirement simultaneously in a short time. Hence, I separated each two numbers of the remaining 4 rows of the matrixes above-mentioned, and readjusted the program to search. However, because there were numerous combinations of permutation, the program ran for several days and could not find the solution meeting the requirement of all 16 rows.

I pondered hard about the restrictive conditions of the program. Then I found when the above rows were adjusted, the algorithm fetching numbers in the same column could be improved to fetch numbers in the same column or in the adjoining columns. More matrixes meeting the requirement of all 12 rows simultaneously came out. Owing to the increasing amount of calculation, several computers worked together for some days, but the program was still running. Though there were a lot of matrixes meeting the requirement of all 12 rows simultaneously, which added up to some hundred thousand, there was no one meeting the requirement more than 12 rows simultaneously.

The breakthrough happened when Prof. Chen analyzed my results and made some very important discoveries. In the course of my communicating with Prof. Chen, he took my results which I obtained to analyze. When he knew I obtained the solution which could meet the requirement of 12 rows simultaneously and the sum, the sum of square of the remaining 4 rows could meet the requirement, he thought the result was significant and it had approached the advanced level of research of 16th magic square in the world.

During the seven-day Labor Day vacation, Prof. Chen and I kept on researching without interruption. This time Prof. Chen took the data to analyze and the next day he entered the campus to prove his vital discovery, and he told me his discovery: If 8 pairs of combination numbers meet the requirement, they must be 4 pairs of even numbers and 4 pairs of odd numbers, and the sum, square sum, cubic sum of the 4 pairs of even and odd number combination are constant values. (The sum is 1016 or 1024, the sum of square is 172720 or 174760, and the sum of cubic is 33032192 or 33553408)

I was deep in thought again. If Prof. Chen's discovery was right, the amount of calculation would

decrease considerably. The next day, I programmed to prove that the discovery was right. And I also discovered another important formula: If the sum, the sum of square and the sum of cubic of the 4 pairs of even number combination are constant values mentioned above, add 1 to every even number and it will be transformed to 4 pairs of odd number combination, whose sum, square sum and cubic sum meet the constant mentioned above. In this way, the amount of programming to construct the magic square could be decreased to a half. Then I readjusted the program again, and soon I gained a matrix that had 14 rows meeting the requirement simultaneously, there were only two rows left to the final result.

Isn't there a whole solution to the problem containing all characteristics? Though it was Labor Day vacation, I was weighed down with researching trimagic square. I continued studying 20th, 24th, 28th, 32nd trimagic square. The 24th had 22 rows meeting the requirement simultaneously, and such results showed by the computer were innumerable as the hair on an ox, we could find millions of matrixes like that with computer, but the whole solution to the 24th had not been found yet. The 32nd had 32 rows meeting the requirement for some time, but when the columns were adjusted, because of its huge calculation, the result still could not be found.

I was not reconciled to the fact! I considered 16th trimagic square over again. Because the calculation was huge, I made the algorithm search the result only in the same column or in the adjoining columns. However, because we could execute the program quickly now, we could surely make out an algorithm to search the whole matrix to prove the existence of its solution.

That afternoon, I rewrote the program. In spite of its complexity, I kept on thinking hard. As I had not had a good rest, I felt so tired that my head ached seriously. But I was still considering the problem while I was taking a rest. Finally the program was executed successfully, and the result came out soon. Then I checked the result. Eureka! All the 16 rows met the requirement, and I couldn't control my excitement. As an ancient Chinese poem goes:

I look for her for million times. When all at once I turn my head, I find her there where lantern light is dimly shed. (An ancient Chinese poem written by Xin Qiji)

After adjusting the 16 rows, it's as easy as winking to adjust 16 columns and the diagonals. However, as I didn't take a rest at noon, I was still afflicted by headache. Then I thought I needed a good rest.

In the evening, after a rest of several hours, I felt a little better, and I went on arranging the 16 columns of the matrix. The following morning, when I told Prof. Chen the exciting news, he was so happy that he couldn't helps jumping. He came to the campus in the morning against his own rule, and we immediately wrote a program to search the diagonals.

Those who work hard will be rewarded！A month of painstaking work had not been done for nothing. After working day and night and surmounting so many difficulties, we got a perfect result at last. 16th trimagic square many people search for you even in their dreams, when you come to us quietly, you look as beautiful as beautiful can be!

Learning the news that 16th trimagic square came out, Gao Zhiyuan, Li Kangqiang, chairmen of Chinese Magic Square Pursuers Association , and a French friend sent me letters of congratulation.

Here are some of them:

Dear Mr. Chen Qinwu:

Learning of your success in searching for the 16th-trimagic square, I feel very happy! The solution of 16th trimagic square is an important achievement. Many pursuers have studied for years, and we have been longing for the result even in our dreams. Now you and Prof. Chen Mutain succeeded in working out the solution to the 16th trimagic with computer through your efforts and operation. What inspiring news! You have won honor for us Chinese. I congratulate you on behalf of the Chinese Magic Pursuers Association.

We hope you to publish this result as soon as possible. Of course, we hope it can be certificated internationally. We have a French friend, who is collecting the results of magic square all over the world. I believe your success can boost our own morale.

Gao Zhiyuan, Yan'an Education College.

Dear Mr. Chen Qinwu：

Hello！

16th trimagic square is really a stronghold hard to capture. Seven years have elapsed, since 1997. Many magic square expert have made painstaking efforts in it， and you have succeeded. You have won honor for us Chinese， I, myself, on behalf of friends of Chinese Magic Pursuers Association, congratulate you and your partner sincerely, and hope you to achieve more successes. I wish the result can be certificated internationally.

Now Mr. Wang Zhonghang is taking charge of the accounts of our association, I hope we can cash the premium set for this. Of course, you have studied magic square not for this little premium, but we must keep our promises.

We have received and saved all the mails you sent. We will publish them in our publication.

Li Kangqiang, Chairman of Chinese Magic Square Pursuers Association.

Dear friends,

I have checked your square, and yes it is a trimagic square. Congratulations!

Your square has been added in the update done today of www.multimagie.com/indexengl.htm

Click on News of May 2005 in the left menu.

I understand, with your email addresses, that you are working at the Shantou University.

Are you students or teachers? Could you explain how you have constructed your square?

Best regards from France.

Christian Boyer.

Translated by Prof. Zhuang Hecheng

Chen Junwei

Zhang Quan

Attachment:

16th Trimagic Square

by Chen Qinwu, Chen Mutian for 2005/5/8

(Computer Science Lab. Shantou University. qwchen@stu.edu.cn)

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33	29	27	25	145	82	84	114	141	171	173	110	230	228	226	222	2040	347480	66585600
51	39	123	63	233	109	206	218	37	49	146	22	192	132	216	204	2040	347480	66585600
177	167	225	211	168	244	150	41	214	105	11	87	44	30	88	78	2040	347480	66585600
124	200	4	248	111	90	48	102	153	207	165	144	7	251	55	131	2040	347480	66585600
195	179	175	231	198	58	95	240	15	160	197	57	24	80	76	60	2040	347480	66585600
61	77	81	117	246	213	113	14	241	142	42	9	138	174	178	194	2040	347480	66585600
202	252	106	126	96	43	12	101	154	243	212	159	129	149	3	53	2040	347480	66585600
118	54	70	188	209	235	19	163	92	236	20	46	67	185	201	137	2040	347480	66585600
254	98	184	66	65	75	237	93	162	18	180	190	189	71	157	1	2040	347480	66585600
136	156	250	128	23	181	170	17	238	85	74	232	127	5	99	119	2040	347480	66585600
130	134	182	186	8	172	35	239	16	220	83	247	69	73	121	125	2040	347480	66585600
52	2	148	68	191	147	242	155	100	13	108	64	187	107	253	203	2040	347480	66585600
223	227	229	139	158	196	143	36	219	112	59	97	116	26	28	32	2040	347480	66585600
0	120	72	6	47	164	161	152	103	94	91	208	249	183	135	255	2040	347480	66585600
79	89	31	45	86	10	104	215	40	151	245	169	210	224	166	176	2040	347480	66585600
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Other Website: http://cslab.stu.edu.cn http://cboyer.club.fr/multimagie http://www.zhghf.com

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