Mathematical Puzzles, some of the first number puzzles were included in an important ancient Egyptian mathematical document composed about 1650 BC and known as the Rhind Papyrus. Magic squares, another early form of number puzzle, originated in China before the end of the 1st century. A magic square puzzle forms a square array of numbers so that the rows, columns, and major diagonals all have equal sums.

Magic Square, in mathematics, array of distinct numbers so arranged in a square that the sums of each row, each column, and each main diagonal are equal. For example, the square array

$$\begin{matrix} 2 & 7 & 6 \\ 9 & 5 & 1 \\ 4 & 3 & 8 \end{matrix}$$

is a magic square of order 3 (the order is the number of horizontal rows or vertical columns), the constant sum of which is 15. In ancient times such configurations of numbers were thought to be good-luck charms or talismans. Later, mathematicians became interested in the magic square as a problem in mathematical analysis.

The numbers of a magic square of order n are almost always restricted to the integers 1, 2, 3, ..., n^2. The sum of

$$1, 2, 3, \ldots, n^2 \text{ is } \frac{n^2(n^2+1)}{2}$$

Therefore, the sum of each of the n rows, each of the n columns, or each of the two main diagonals of the magic square is

$$\frac{n(n^2+1)}{2}$$

This number is called the magic-square constant. In addition to the main diagonals, which in the illustration above are the triads (2,5,8) and (6,5,4), one could also consider the broken diagonals, which here are (7,1,4); (6,9,3); (2,1,3); and (7,9,8). A magic square is called panmagic or pandiagonal if the sum of each broken diagonal is also equal to the magic-square constant. The magic square of order 3 shown above is not panmagic, but the magic square of order 4:

$$\begin{matrix} 1 & 8 & 10 & 15 \\ 12 & 13 & 3 & 6 \\ 7 & 2 & 16 & 9 \\ 14 & 11 & 5 & 4 \end{matrix}$$

is panmagic because the numbers in each of the four rows, four columns, and eight diagonals all add up to 34.

A magic square is called bimagic or doubly magic if it remains a magic square when each element is replaced by its square; it is called trimagic or triply magic if it remains a magic square when each element is replaced by its square and its cube.

A magic square, with elements 1,2, ..., n^2, exists for every order n except n = 2; to date, however, no general rule has been found for constructing all magic squares, and it is not known how many distinct magic squares exist for every order n. Rules for the construction of particular magic squares have been developed for three classes: Those for which the order n is odd, those for which the order n is divisible by 2 but not by 4, and those for which the order n is divisible by 4. Magic cubes and other magic geometrical figures have also been studied.

The Latin square, a distant relative of the magic square, is a square having as elements the integers 1,2, ..., n (or any n distinct numbers), each occurring n times and arranged so that the integers in any row or column are distinct. Thus

$$\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{array} \qquad \begin{array}{ccc} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{array}$$

are Latin squares. If the second is superposed on the first, maintaining order, to form the square of pairs

$$\begin{array}{ccc} 1,1 & 2,2 & 3,3 \\ 2,3 & 3,1 & 1,2 \\ 3,2 & 1,3 & 2,1 \end{array}$$

we note that no pair is repeated. Such a square of pairs, without repetition of pairs, is called a Euler square, after the Swiss mathematician Leonhard Euler, or a Greco- (or Graeco-) Latin square. Latin and Euler squares have received considerable attention.

Credit: Berggren, J. Lennart, and Singer, James. "Magic Square." Microsoft® Encarta® 2009 [DVD]. Redmond, WA: Microsoft Corporation, 2008.